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The most significant research progress and accomplishment in our project is on the modelling, analysis and designs of stabilizing joints for coupled structures. In a series of papers, we have studied second order dynamic structures modelling vibrating strings and cables, and fourth order dynamic structures modelling vibrating beams. We are able to classify all linear dissipative joints into types. Dr. H.H. West of the Civil Engineering Department of the Pennsylvania State University has collaborated with us and completed mechanical designs for all of them.

Numerical and experimental verification have also been carried out. Dr. Chen's Ph.D. student M.P. Coleman is now using the Cyber 205 supercomputer to compute eigenfrequencies of a damped plate. Experiments were conducted at the MIPAC Facility of the University of Wisconsin in collaboration with Dr. D.L. Russell.

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FINAL PROJECT REPORT

TO

THE AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

FOR

AFOSR Grant 85-0253

"Stabilization and Control Problems in Structural Dynamics"

September 1, 1985 - August 31, 1987

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[illegible]			

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Final Project Report of AFOSR Grant #85-0253

September 1985-August 1987

I. Introduction.

During the Support period of AFOSR Grant #85-0253, September 1985 through August 1987, Dr. G. Chen, the principal investigator, and Dr. J. Zhou, the co-investigator, together with their collaborators and doctoral students have written a number of papers (cf. [1]-[13] in §III List of Publications) and have given presentations at meetings and technical briefings. The primary emphasis of the research project is on the *modelling, designs, placements, analysis* and *computations* of stabilizers and controllers in structural dynamics. The cumulative research achievements and activities are summarized below.

II. Research Accomplishments.

We first describe our research findings under headings [A]-[I] below:

[A] **Analysis and designs of stabilizers for second order systems-vibrating strings and cables** (cf. [3]).

The standard model is the wave equation

$$m\frac{\partial^2 y(x,t)}{\partial t^2} - T\frac{\partial^2 y(x,t)}{\partial x^2} = 0, \qquad 0 < x < L, \quad t > 0.$$

where $y(x,t)$ represents the vertical displacement at position x at time t, m is the mass density per unit length, and T is the tension coefficient. The energy of vibration at time t is

$$E(t) = \frac{1}{2}\int_0^1 [my_t^2(x,t) + Ty_x^2(x,t)]dx.$$

If a stabilizer is placed at the boundary $x = L$, then

$$\frac{d}{dt}E(t) = Ty_t(L,t)y_x(L,t) \leq 0, \tag{1}$$

assuming that the boundary condition at $x = 0$ is energy conserving. For (1) to hold, we must use proportional control

(2) $\quad Ty_x(L,t) = -k^2 y_t(L,t), \qquad k^2 = \text{feedback gain} > 0,$ for all $t > 0$, *i.e.*, force is negatively proportional to velocity at $x = L$.

Therefore a standard viscous damper is sufficient for this purpose, as shown below:

Fig. 1. A vibrating string with a damping device at left end

The above fact is well known. Furthermore, if one chooses the feedback gain to be

$$k^2 T\sqrt{T/m},$$

then the damper action (2) becomes a so-called characteristic impedance condition, which causes maximum energy loss to the vibrating system.

Now, let us consider the case when the stabilizer is installed at an in-span point, say at $x = \alpha, 0 < \alpha < L$. Assume that the boundary conditions at $x = 0$ and $x = L$ are energy-conserving. Then

(3) $$\frac{d}{dt}\,[\text{Total energy at time t}] = \frac{d}{dt}\Big\{\frac{1}{2}\int_0^\alpha [my_t^2(x,t) + Ty_x^2(x,t)]dx - \frac{1}{2}\int_\alpha^L [my_t^2(x,t) + Ty_x^2(x,t)]dx\Big\} = Ty_x(\alpha^-,t)y_t(\alpha^-,t) - Ty_x(\alpha^+,t)y_t(\alpha^+,t) \le 0.$$

There are only two possibilities which can cause the negativity of the right hand side of (3):

(A1) Continuity of displacement (and velocity)

$$y(\alpha^-,t) = y(\alpha^+,t)$$

discontinuity of vertical force

$$Ty_x(\alpha^-,t) - Ty_x(\alpha^+,t) = -k_1^2 y_t(\alpha^-,t), \quad k_1^2 = \text{feedback gain} > 0;$$

(A2) Discontinuity of displacement and velocity

$$y_t(\alpha^-,t) - y_t(\alpha^+,t) = k_2^2 Ty_x(\alpha^-,t), \quad k_2^2 = \text{feedback gain} > 0,$$

continuity of vertical force

$$Ty_x(\alpha^-,t) = Ty_x(\alpha^+,t).$$

The mechanical designs of stabilizers of the above two types are given in Figures 2 and 3.

The mathematical analysis of damping behavior associated with the above designs is given in our paper [3], along with numerical results.

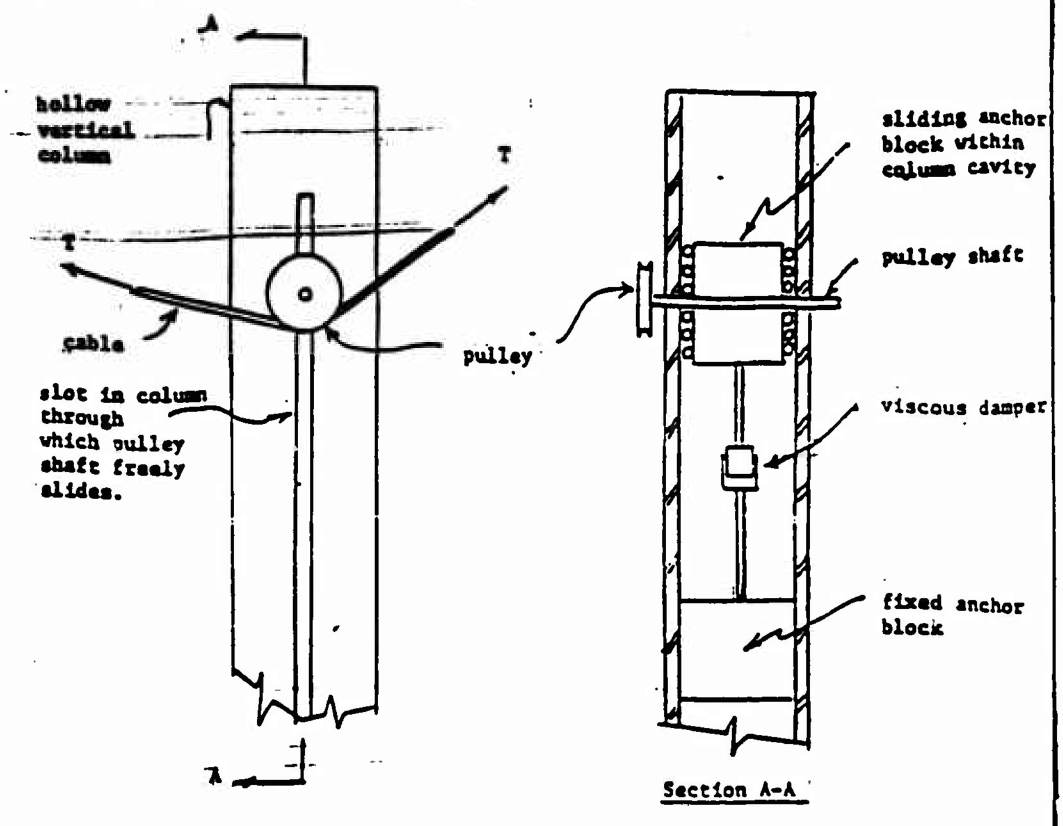

Fig. 2. Stabilizer device satisfying (A1)

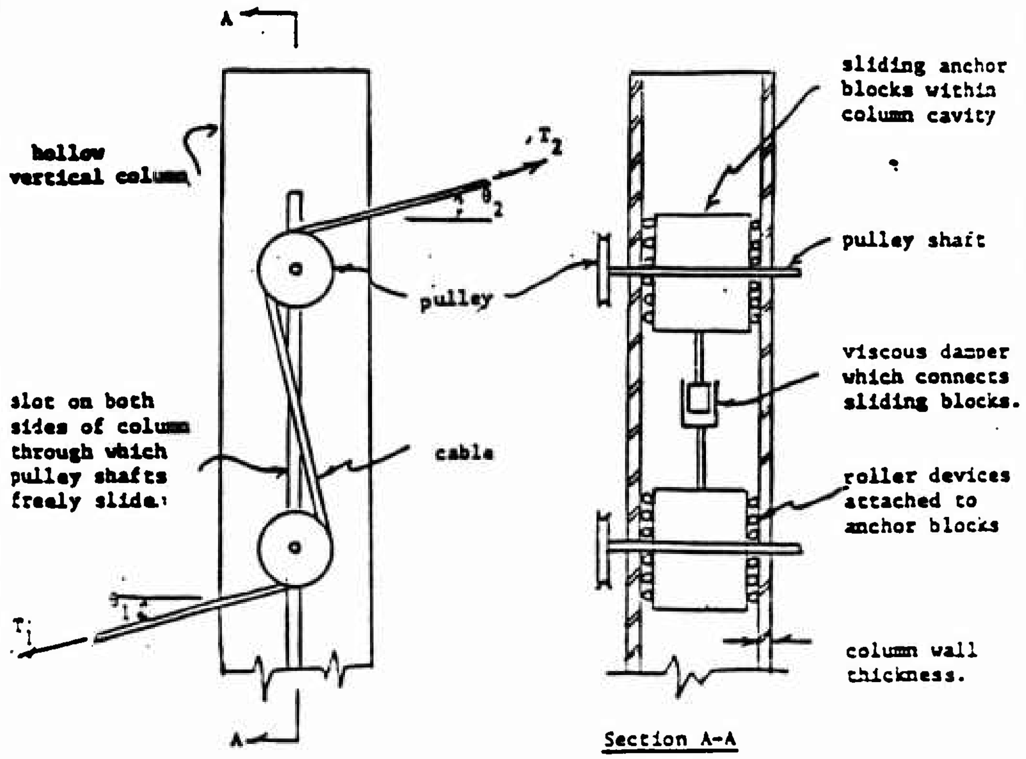

Fig. 3. Stabilizer device satisfying (A2)

[B] Stabilization and Control of Serially Connected Beams (cf. [1], [6])

A long Euler-Bernoulli beam of length L is formed by connecting many segment beams. One end of the beam is clamped, and the other end is free. At the connection joints and at the free end, there are co-located actuators and sensors to stabilize the vibration of the beam. The configuration and mathematical modeling of joint conditions are indicated in Figure 4:

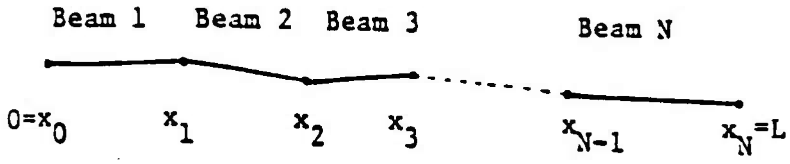

Fig. 4. Serially connected beams

Each i-th beam satisfies

$$m_i \frac{\partial^2 y}{\partial t^2} + E_i I_i \frac{\partial^4 y}{\partial s x^4} = 0, \quad x_{i-1} < x < x_i, \quad t > 0, 1 \le i \le N.$$

The beam is clamped at $x = 0$:

$$y(0,t) = 0, \frac{\partial y}{\partial x}(0,t) = 0, t > 0.$$

At each intermediate node x_i, the beam is strongly connected:

$$y(x_i-,t) = y(x_i+,t), \frac{\partial y}{\partial x}(x_i-,t) = \frac{\partial y}{\partial x}(x_i+,t) \quad t > 0$$

with a point force $u_{0i}(t)$ and a point bending moment $u_{1i}(t)$ applied at x_i :

$$\left.\begin{aligned} E_i I_i \frac{\partial^3 y}{\partial x^3}(x_i-,t) - E_{i+1} I_{i+1} \frac{\partial^3 y}{\partial x^3}(x_i+,t) &= u_{0i}(t) \\ -[E_i I_i \frac{\partial^2 y}{\partial x^2}(x_i-,t) - E_{i+1} I_{i+1} \frac{\partial^2 y}{\partial x^2}(x_i-,t)] &= u_{1i}(t) \end{aligned}\right\} \qquad 1 \le i \le N-1$$

At the end $x_N = L$, velocity and angular velocity are (positively and negatively) proportionally used as stabilizing feedback:

$$\left\{ \begin{aligned} E_N I_N \frac{\partial^3 y}{\partial x^3}(L,t) &= k_0^2 \frac{\partial y}{\partial t}(L,t) \\ E_N I_N \frac{\partial^2 y}{\partial x^2}(L,t) &= -k_1^2 \frac{\partial^2 y}{\partial x \partial t}(L,t) \end{aligned} \right\} \qquad k_0^2, k_1^2 > 0$$

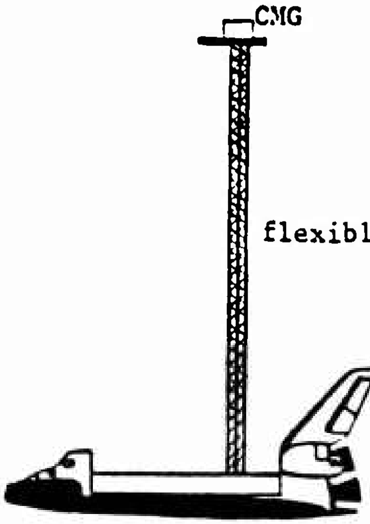

Fig. 5. Spacecraft mast control experiment

The mathematical theory is intended to model the *mast control system* in NASA's *COFS* and *SCOLE Programs.* (See Figure 5). A long flexible mast of length 60 meter is clamped to its base on a space shuttle. The mast is formed with 54 bays and could be considered a segmented beam. At the very end of the mast, a *CMG* (control moment gyro) is placed which can apply bending and torision control to the mast according to sensor feedback. The contractor for this mast control system is Harris Corporation in Melbourne, Florida. After preliminary discussions with Harris engineer Dr. D.C. Hyland in Melbourne in early August, we agree that the feedback control law (4) reflects the basic features of the CMG mast control system to be used for COFS.

The main result of our paper [1] proves that with the application of the feedback law (4), the energy of vibration of the beam decays uniformly exponentially:

$$E(t) \leq K e^{-\mu t} E(0), \; K > 0, \; \mu > 0.$$

Therefore the system can be controlled and stabilized.

The theory and methods in our work can be extended and generalized to model and control more elaborate (high performance) control system in future large flexible space structures.

[C] Stability criterion for point stabilizers on coupled quasilinear vibrating strings (cf. [7]).

Consider two quasilinear wave equations

$$
(5)\qquad
\begin{aligned}
&\frac{\partial^2}{\partial t^2}w(x,t) - \frac{\partial}{\partial x}\left[\sigma_1\left(\frac{\partial w(x,t)}{\partial x}\right)\right] = 0, \qquad 0 < x < 1,\\
&\frac{\partial^2}{\partial t^2}w(x,t) - \frac{\partial}{\partial x}\left[\sigma_2\left(\frac{\partial w(x,t)}{\partial x}\right)\right] = 0, \qquad 1 < x < 2.
\end{aligned}
$$

where σ_1 and σ_2 are nonlinear functions satisfying $\sigma_i(0) = 0, \sigma_i(u) > 0$, for $i = 1,2$. The boundary condition at $x = 0$ is

$$
(6)\qquad \sigma_1(w_x(0,t)) - k_0^2 w_t(0,t) = 0, \quad k_0^2 > 0,
$$

and at $x = 2$ is

$$
(7)\qquad w(2,t) = 0.
$$

The two strings are coupled at $x = 1$ through

$$
(8)\qquad
\begin{aligned}
&\sigma_1(w_x(1^-,t)) = \sigma_2(w_x(1^+,t))\\
&w_t(1^-,t) - w_t(1^+,t) = -k_1^2\sigma_1(w_x(1^-,t)), \quad k_1^2 \geq 0.
\end{aligned}
$$

Using some fairly recent theorems on quasilinear hyperbolic PDEs, we are able to show that if k_0^2 and k_1^2 in (6) and (8) are nonnegative, then the solution decays exponentially in the C^1-norm:

(9) $$\|w(\cdot,t)\|_{C^1(0,1)} + \|w(\cdot,t)\|_{C^1(1,2)} \le Ke^{-\alpha t}, K, \alpha > 0 \quad \text{for all } t \ge 0,$$

provided that

$$\begin{aligned} 1 + K_1K_3 + K_2 + K_1K_2K_3 - 4c_1c_2\Delta^{-2}K_3 &> 0 \\ 2(1 - K_1K_2K_3 + 4c_1c_2\Delta^{-2}K_3) &> 0 \\ 1 - K_1K_3 - K_2 + K_1K_2K_3 - 4c_1c_2\Delta^{-2}K_3 &> 0 \end{aligned}$$

are satisfied, where

$$\begin{aligned} c_i &\equiv \sqrt{\sigma_1(0)}, \quad i = 1, 2, \quad c_1 \ge c_2, \\ \Delta &\equiv c_1 + c_2 + k_1^2 c_1 c_2 \\ K_1 &\equiv \Delta^{-1}(c_1 - c_2 + k_1^2 c_1 c_2) \\ K_2 &\equiv \Delta^{-1}|c_1 - c_2 - k_1^2 c_1 c_2| \\ K_3 &= |c_1 - k_0^2|/(c_1 + k_0^2). \end{aligned}$$

for solutions of (5)-(8) with sufficiently smooth small initial data. Note that if $k_0^2 = 0$ in (6), then generally (9) does not hold no matter how $k_1^2 > 0$ is chosen. This shows **that installing a stabilizer in the middle of the span (without installing another one at the** boundary) is *not robust* with respect to stability.

[D] The boundary element methods for optimal boundary control of elliptic partial differential equations (cf. [2]).

Elliptic PDEs often appear in continuum mechanics. Assume that the control appears on the boundary. Traditional numerical methods like the finite differences or finite elements require the discretization of the entire domain. The boundary element method only requires the discretization of the boundary which yields numerical solutions of optimal boundary control much more efficiently. In [2], we treated the Neumann type boundary control using the fundamental solution. In [5], Dirichlet boundary controls were studied using a different Poisson representation of harmonic functions. Numerical solutions were computed which confirmed the theoretical error estimates.

[E] A Boundary element method for the linear quadratic problem based on cauchy integrals (cf. [5]).

In a given multi-connected domain,

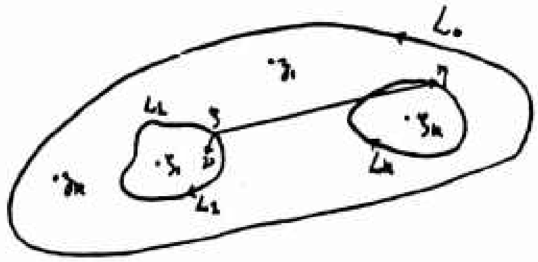

Fig. 6. A multi-connected domain with finitely many sensors.

consider the Dirichlet control problem for the Laplace equation

$$\inf_{u \in \Gamma_1} \sum_{k=1}^{n} |w(z_k) - \alpha_k|^2 + \beta \int_{\Gamma_1} u^2(x,y)\, d\sigma$$

$$\begin{cases} \Delta w = 0 \\ w|_{\Gamma} \backslash L_0 = 0 \\ w|_{L_0} = u, \end{cases}$$

where $\alpha_1, \ldots, \alpha_n$ are given, $z_1, \ldots, z_n \epsilon C$ are the locations of sensors. The above models some incompressible flows problems or equations in elasticity.

By complex analysis, w can be obtained as Cauchy integrals below

$$\begin{aligned} w(x,y) &= w(z) \\ &= \sum_{j=1}^{M} A_j \ln|z - \varsigma_j| + w^*(z) \\ w^*(z) &= Re\, \phi^*(z) \end{aligned}$$

with

$$\phi^*(z) = \frac{1}{2\pi i} \oint_{\Gamma} \frac{\mu(\varsigma)}{\varsigma - z} d\varsigma,$$

where $\mu(\varsigma)$ is related to the control u through a system of Fredholm (boundary) integral equations of the second kind on Γ :

$$\mu(\eta) - \frac{1}{\pi}\int_\Gamma \frac{\cos(\nu, \eta - \varsigma)}{|\varsigma - \eta|}\mu(\varsigma)d\sigma = 2u(\eta) - 2\sum_{k=1}^{M} A_k \ln|\eta - \varsigma_k|$$

on L_0;

$$\mu(\eta) - \frac{1}{\pi}\int_\Gamma \frac{\cos(\nu, \eta - \varsigma)}{|\varsigma - \eta|}\eta(\varsigma)d\sigma - \frac{1}{\pi}\int_{L_k} \mu(\varsigma)d\sigma = -2\sum_{j=1}^{M} A_j \ln|\eta - \varsigma_j|$$

on $L_k, k = 1, ..., M$ along with

$$\int_{L_k} \mu(\varsigma)d\sigma = 0, \quad k = 1, ..., M.$$

The above forms a system of $M+1$ integral equations and M linear algebraic equations. Again, a reduction of space dimension is effected so it gives an economical way to compute the optimal control.

We are presently simplifying the algorithms for general domains. We are also comparing the efficiency of this approach with the conventional boundary element method.

[F] **Analysis, modelling, designs and experiments of dissipative joints for co-linearly coupled beams (cf. [9], [11]).**

In the construction of modern large flexible space structures, active and passive damping devices are commonly installed at joints of coupled beams to achieve the suppression of vibration. In order to successfully control such dynamic structures, the function and behavior of dissipative joints must be carefully studied.

We analyze these dissipative joints by first classifying them into types according to the discontinuities of physical variables across a joint. The four important physical variables for beams are displacement (y), rotation (θ), bending moment (M) and shear (V). We can classify dissipative joints into the following four types:

(1)	M	*and*	V	*are continuous,*	y	*and*	θ	*are discontinuous;*
(2)	y	*and*	M	*are continuous,*	θ	*and*	V	*are discontinuous;*
(3)	y	*and*	θ	*are continuous,*	M	*and*	V	*are discontinous;*
(4)	θ	*and*	V	*are continuous,*	y	*and*	M	*are discontinous,*

according to the conjugacy of these variables. We have achieved mechanical designs for all these dissipative joints of the linear passive type.

The spectrum of two identical coupled beams with a linear dissipative joint shows an interesting pattern. We prove that there are two families of eigenvalues, asymptotically appearing alternately and parallel to the imagnary axis with eigenfrequencies spaced vertically with gap $0(n^2)$.

This interesting spectral behavior has also been observed and studied in experiments conducted at the MIPAC Facility of the University of Wisconsin. Numerical simulations using the Legendre spectral method has also confirmed the same property.

The experimental data have been smoothed by a least square optimization procedure based upon the theoretical estimates. The smoothed data show a very regular spacing property of the eigenfrequencies in support of our modelling and theory.

[G] **Boundary control for a thin isotropic homogeneous elastostatic plate using the boundary element method** (cf. [10]).

In recent development of adaptive optics, the problem of shape control of a reflecting mirror is studied. Assume that the mirror is modelled by a thin elastostatic plate. At certain interior points of the plate a number of sensors are located which measure the deformation at those points. We wish to apply boundary controls to the plate so that the sensory data are as close to the prescribed values as possible. In this paper we present a boundary element method to approximate optimal boundary controls for a quadratic cost problem. The method has been tested to have high accuracy and efficiency. Numerical results are also presented.

[H] **The boundary element method for the Helmholtz equation in scattering** (cf. [13]).

We consider the problem of minimizing the scattered field intensity with respect to the boundary impedance. Usual optimization procedures based on gradient type local mimization algorithms will not be effective for this problem because the scattered field intensity is not a convex function of the impedance, and it has many local extrema.

We approach the problem here by first discretizing the Helmholtz partial differential equation using the boundary element method. Then we apply some recently developed global optimization algorithms to find approximate distributions of the boundary impedance for particular shapes which minimize the reflected field intensity. The boundary element method effects a reduction of dimensionality resulting in much greater computational efficiency. The global optimization algorithm allows us to pick out nearly global minimum solutions among many local minima.

Numerical solutions are represented graphically and discussed. Our results show that a variable boundary impedance is much effective for minimizing the scattered field than a constant boundary impedance.

The methods have potential applications to the design of optimal reflective coating for a stealth surface.

[I] **Existence of "social equilibrium" for games subject to inequality or linear equality constraints** (cf. [12]).

We consider the existence of "social equilibrium" strategies for a two person nonzero sum game with inequality or equality constraints. The two players must coordinate their choices of strategies to make the combined strategies admissible, i.e., satisfy the given constaints.

We approach the problem by first converting it into a quasi-variational inequality (QVI). We then apply our recently proved theorem on the existence of solutions of QVI to determine appropriate properties of the inequality constraints in order to guarantee the existence of a social equilibrium for the given game. If the equality constaints are linear and affine, we show that such existence will always hold.

We apply our theorems to establish existence of an open loop equilibrium of a constrained differential game to illustrate the theory.

III. List of Publications

The following is a list of publications supported by the grant during September 1, 1985-August 31, 1987. Two preprint copies of each paper have been sent to the Program

Manager. When they appear in journals in final form, reprints will be submitted to AFOSR immediately.

[1] G. Chen, M.C. Delfour, A.M. Krall and G. Payre, Modelling, stabilization and control of serially connected beams, SIAM J. Control Opt. 25 (1987), 526-546.

[2] G. Chen and Y.L. Tsai, The boundary element numerical method for two dimensional linear quadratic elliptic problems: (I) Neumann control, Math. Comp., 44 (1987), to appear.

[3] G. Chen, M. Coleman, and H.H. West, Pointwise stabilization in the middle of the span for second order systems, nonuniform and uniform decay results, SIAM J. Appl. Math. 47 (1987), 751-780.

[4] G. Chen and J.X. Zhou, Diagonal convexity conditions for problems in convex analysis and quasi-variational inequalities, accepted, to appear in J. Math. Anal. Appl..

[5] G. Chen, C.P. Chen and I. Aronov, A boundary element method based on Cauchy integrals for some linear quadratic boundary control problems on a circle, Optimal Control Applications & Methods, 8 (1987), to appear.

[6] G. Chen, S.G. Krantz, D.W. Ma, C.E. Wayne, and H.H. West, The Euler-Bernoulli beam equation with boundary energy dissipation, pp. 67-96, in "Operator Methods for Optimal Control Problems", S.J. Lee, ed., Lecture Notes in Pure and Applied Mathematics Series, Marcel Dekker, Inc., New York.

[7] G. Chen and H.K. Wang, Pointwise stabilization for coupled quasilinear and linear wave equations, presented at the Conference on Control and Identification of Distributed Systems, Vorau, Austria, July 1986. To appear in Springer Lecture Notes on Control and Information Sciences.

[8] H.K. Wang and G. Chen, Asymptotic behavior of solutions of the one-dimensional wave equations with a nonlinear elastic dissipative boundary condition, preprint, in review.

[9] G. Chen, S.G. Krantz, D.L. Russell, C.E. Wayne, H.H. West and M.P. Coleman, Analysis, designs and behavior of dissipative joints for coupled beams, preprint, in review.

[10] G. Chen and J. Zhou, Computing optimal boundary controls of a plate by the bound-

ary element method, to appear in the Proceedings of the 26th IEEE CDC, Los Angeles, CA, December 1987.

[11] G. Chen, S.G. Krantz, D.L. Russell, C.E. Wayne, H.H. West and J. Zhou, Modelling, analysis and testing of dissipative beam joints - experiments and data smoothing, presented at the 6th International Conference on Mathematical Modelling, Washington Univ., St. Louis, MO, Aug. 1987. To appear in Conference Proceedings, Pergamon Press, New York, 1988.

[12] J. Zhou, Existence for social equilibrium of nonzero sum games with inequality or linear equality constraints, preprint, in review.

[13] G. Chen, T.J. Bridges and J. Zhou, Minimizing the reflection of waves by surface impedance using boundary elements and global optimization, preprint, in review.

IV. Ph.D. Theses Supported by the Grant.

(a) Mr. J. Zhou completed his Ph.D. thesis entitled "Topics in Differential Games and Variational Inequalities" under the advisement of Dr. G. Chen in August 1986. The theorems he proved in the thesis have potential applications to equilibrium PDE problems and multi-objective optimization.

Dr. Zhou is presently an assistant professor in the Department of Mathematics of Texas A&M University, College Station, Texas.

(b) Mr. H.K. Wang completed his thesis entitled "Study of Stabilization and Energy Dissipation for Second Order Vibrating Systems" under the advisement of Dr. G. Chen in August 1987, with partial support from this grant.

Mr. Wang has accepted a tenure-track assistant professorship at the Wichita State University Wichita, Kansas, to begin August 1987. He is expected to work jointly with several aerospace engineering faculty in the newly established Aviation Institute at that university.

Three copies of the doctoral theses have been sent to the Program Manager.

V. Data on Scientific Collaborators.

The following is a list of collaborators and their affiliations:

1. I. Aronov, Graduate Student, now at Cornell University.

2. T.J. Bridges, Assistant Professor, Mathematics Department, Worcester Polytechnic Institute, Worcester, MA.
3. C.P. Chen, Graduate Student, The Pennsylvania State University.
4. M. Coleman, Graduate Student, The Pennsylvania State University.
5. M.C. Delfour, Professor, Universite' de Montreal.
6. A.M. Krall, Professor, The Pennsylvania State University
7. S.G. Krantz, Professor, Washington University, St. Louis, MO
8. D.W. Ma, Visitor, The Pennsylvania State University
9. G. Payre, Assistant Professor, Chemical Engineering Department, Universite' de Sherbrooke
10. D.L. Russell, Professor, Mathematics Department, University of Wisconsin, Madison, WI
11. Y.L. Tsai, Graduate Student, The Pennsylvania State University
12. C.E. Wayne, Associate Professor, The Pennsylvania State University
13. H.K. Wang, Ph.D. Student. now Assistant Professor, Mathematics Department. Wichita State University, Wichita, KS
14. H.H. West, Professor of Civil Engineering, The Pennsylvania State University
15. J.X. Zhou, former Ph.D. Student, now Assistant Professor, Mathematics Department, Texas A&M University

VI. **Activities.**

(a) Sept. 1985-Aug. 1986 (first year)

Dr. Chen gave five invited talks at the following institutions and conferences:

1. Mathematics Department, Georgetown University, Washington, D.C., October 1985.
   Subject matter: point stabilizer for coupled wave equations
   Individual involved: Professor J. Lagnese
2. Mathematics Department. University of Maryland, College Park, MD, November 1985.
   Subject matter: point stabilizer for coupled wave equations
   Individual involved: Professor T.P. Liu

3. Center for Control Science and Dynamical Systems, University of Minnesota, Mineapolis, MN, April 1986.

   Subject matter: theory, designs and applications of point stabilizers

   Individual involved: Professor E. Bruce Lee

4. MIPAC Facility, Mathematics Research Center, University of Wisconsin, Madison, WI, May 1986.

   Subject matter: theory, designs and applications of point stabilizers

   Individual involved: Professor David L. Russell

5. The International Conference on Identification and Control of Distributed Parameter Systems, Vorau, Austria, Organized by the Technical University of Graz, Austria, July 1986.

   Subject matter: pointwise stabilization for coupled quasilinear and linear wave equations

   Individual involved: Professor Karl Kunisch

Dr. Chen also organized a session on distributed parameter systems in the 22nd Annual Meeting of the Society of Engineering Science, held on October, 7, 8, and 9, 1985 at The Pennsylvania State University. He presented a 30 minute lecture on the applciation of the boundary element method to compute optimal controls of certain linear quadratic elliptic problems

(b) Sept. 1986-Aug. 1987 (second year)

Dr. Chen has given talks at the following institutions and conferences:

1. Dynamical Ssytems Conference, Pennsylvania State University, University Park, September 1986.

   Talk title: Asymptotic behavior of solutions of the one-dimensional wave equation with a nonlinear dissipative boundary condition.

   Presented jointly with Mr. H.K. Wang.

2. Mathematics Department, University of Nebraska, Lincoln, NE, October 1986.

   Talk title: Theory, designs and applications of point stabilizers for dynamic structures.

   Individual involved: Professor R. Rebarber.

3. The Second Conference on Control of Distributed Parameter Systems, Val David, Quebec, Canada, October 1987.

   Invited participant (no talk given).

4. Aeroacoustics Branch, NASA Langley Research Center, Hampton, VA, September 1986.

   Talk title: Near field pressure fluctuations in supersonic jets, presented jointly with Prof. P.J. Morris of Aerospace Engineering Department, Pennsylvania State University.

   Individuals involved: Dr. P. Pao, Dr. J. Seiner.

5. Structural Dynamics Branch, NASA Langley Research Center, Hampton, VA, September 1986.

   Talk title: Analysis and designs of beam joints.

   Individual involved: Dr. G. Horner.

6. Mathematical Sciences Section AFOSR, Bolling Air Force Base, D.C., December 1986.

   Technical briefing: Minimizing the reflection of waves by surface impedance using boundary elements and global optimization.

   Individual involved: Dr. J.M. Crowley.

7. Department of Mathematics, Texas A&M University, College Station, Tx February 1987.

   Talk title: Theory, designs and applications of point stabilizers for dynamics structures.

   Individuals involved: Dr. J. Walton, Dr. H.E. Lacey.

8. Department of Mathematics, Iowa State University, Ames, IA. February 1987.

   Talk title: Theory, designs and applications of point stabilizers for dynamic structures.

   Individual involved: Dr. R.K. Miller.

9. Department of Mathematics, Wichita State University, Wichita, KS. February 1987.

   Talk title: Theory, designs and applications of point stabilizers for dynamic structures.

   Invididuals involved: Dr. A. Elcrat, Dr. D. Chopra.

10. Department of Mathematics, Texas A&M University, College Station, TX, March 1987.

Talk title: Uniform exponential decay of energy of evolution equations with locally distributed damping.

Individuals involved: Dr. J. Walton, Dr. J.T. Pitts.

11. The Sixth International Conference on Mathematical Modelling, Washington University, St. Louis, MO, August 1987.

    Talk title: Modelling, designs and analysis of dissipative joints for coupled beams, The presentation was made by Dr. S.G. Krantz of Mathematics Department, Washington University.

    Dr. J. Zhou has given the following two talks:

12. Department of Mathematics, Texas A&M University, College Station, TX, March 1987.

    Talk title: Minimizing the reflection of waves by surface impedance using boundary elements and global optimization.

    Individuals involved: Dr. H.E. Lacey, Dr. J. Walton.

13. Department of Mathematics, Iowa State University, Ames. IA, April 1987.

    Talk title: Minimizing the reflection of waves by surface impedance using boundary elements and global optimization.

    Individuals involved: Dr. R.K. Miller, Dr. K. Heimes.

In addition, Dr. Chen has recently transferred from the Pennsylvania State University to Texas A&M University. His rank has advanced to full professorship. He also holds a joint appointment with the Aerospace Engineering Department. Dr. J. Zhou, formerly postdoctoral fellow, has been appointed an assistant professorship in the Mathematics Department at Texas A&M University.